Hong Kong Mathematics Olympiad (2017/18)

Final Event 1 (Group)

FOR OFFICIAL USE

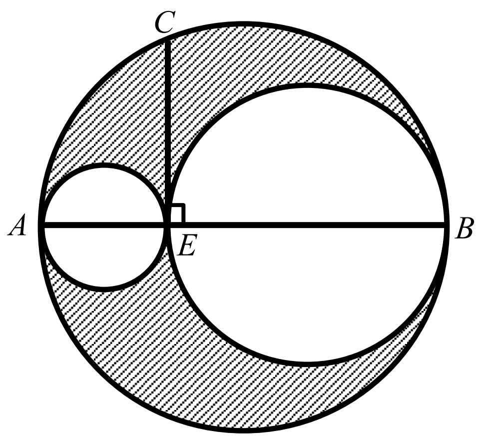
Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	Min.	Sec.
			Total score					

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

1. Suppose that Mary and Ming obtained an integer score either s or t in each of the subjects: Chinese, English and Mathematics, where $s > t > 0$. It is known that Mary did better in Chinese but Ming did better in English. Mary's and Ming's total scores are 12 and 9, respectively. Determine the value of s.
瑪莉和小明在中文科、英文科及數學科獲得的分數為 s 或 t 的整數，且 $s > t > 0$。若瑪莉於中文科的分數比小明的高以及小明於英文科的分數比瑪莉的高，而瑪莉和小明的總分分別是 12 分和 9 分。求 s 的值。

$s =$

2. Given that two circles, one with diameter AE and the other with diameter BE, are inscribed by a large circle with diameter AB. If $CE \perp AB$ with $AB = 10$ and $CE = 4$, and the total area of the shaded regions is $w\pi$, determine the value of w.
已知兩圓的直徑為 AE 及 BE，內接於直徑為 AB 的圓中。若 $CE \perp AB$，$AB = 10$，$CE = 4$ 及著色部份的總面積為 $w\pi$，求 w 的值。



$w =$

3. Let m and r be non-negative integers. If $f(7m + r) = r$, determine the value of $q = f(2^{2018})$.
設 m 及 r 為非負整數。若 $f(7m + r) = r$，求 $q = f(2^{2018})$。

$q =$

4. In base-5 system, if v is the remainder of $234234_5 \div 234_5$, determine the value of v.

在五進制中，若 v 為 $234234_5 \div 234_5$ 的餘數，求 v 的值。

$v =$

Hong Kong Mathematics Olympiad (2017/18)

Final Event 2 (Group)

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=			Team No.		
			+ Bonus score					Time		
			Total score						Min.	Sec.

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除非特別聲明，答案須用數字表達，並化至最簡。

1. Given that $u>0$ and $\dfrac{1-2^{-\frac{1}{u}}}{2^{-\frac{1}{u}}-2^{-\frac{2}{u}}}=4$, determine the value of u .

已知 $u>0$ 及 $\dfrac{1-2^{-\frac{1}{u}}}{2^{-\frac{1}{u}}-2^{-\frac{2}{u}}}=4$ ，求 u 的值。

$u=$

2. Given that $b\geq 1$, $a-12b=15$ and x is a real number, determine the least value of

$v=\dfrac{(x-a)^2}{2b}+5x$.

已知 $b\geq 1$、$a-12b=15$ 及 x 是實數，求 $v=\dfrac{(x-a)^2}{2b}+5x$ 的最小值。

$v=$

3. Suppose that there were 20 boys and 15 girls in a class having taken two tests. Given that 8 students failed the first test, 12 students failed the second test, and 6 students failed both tests, if 5 boys failed the first test, 7 boys failed the second test, 4 boys failed both tests, and n girls passed both tests, determine the value of n .
若班中有 20 位男同學及 15 位女同學參加兩次考試。已知 8 位同學在第一次考試中不合格，12 位同學在第二次考試中不合格，及 6 位同學於兩次考試均不合格。若 5 位男同學在第一次考試中不合格，7 位男同學在第二次考試中不合格，4 位男同學兩次考試均不合格及 n 位女同學兩次考試均合格，求 n 的值。

$n=$

4. Determine the least positive integer m such that $m^{200}>6^{300}$.

求最小正整數 m，使得 $m^{200}>6^{300}$ 。

$m=$

Hong Kong Mathematics Olympiad (2017/18)
Final Event 3 (Group)

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.		
			+ Bonus score				Time		
			Total score					Min.	Sec.

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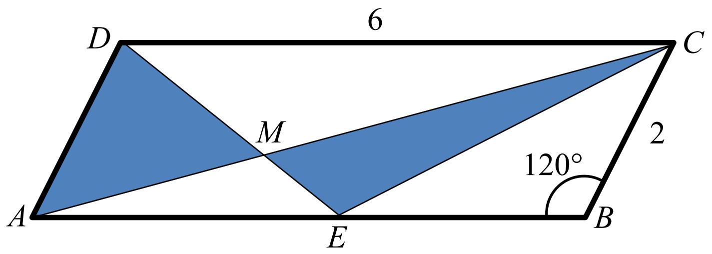
除非特別聲明，答案須用數字表達，並化至最簡。

1. $ABCD$ is a parallelogram with diagonal AC, $CD = 6$, $BC = 2$ and $\angle ABC = 120°$. If E is the midpoint of AB, AC and DE intersect at M, and the total area of the shaded regions is α, determine the value of α.
AC 是平行四邊形 $ABCD$ 的對角線，$CD = 6$，$BC = 2$ 及 $\angle ABC = 120°$。若 E 是 AB 的中點，AC 與 DE 相交於 M 及著色部分的總面積是 α，求 α 的值。



$\alpha =$

2. If β is a 3-digit positive integer that is divisible by 11 and whose quotient when divided by 11 is 3 times the sum of its digits, determine the largest value of β.
設 β 為三位正整數且能被 11 整除，且其商相等於其值的各數字之和的三倍，求 β 的最大值。

$\beta =$

3. Determine the largest real value of φ such that the inequality $\sqrt{1-\varphi}-\sqrt{1+\varphi} \geq 1$ holds.
求 φ 的最大實數值，使不等式 $\sqrt{1-\varphi}-\sqrt{1+\varphi} \geq 1$ 成立。

$\varphi =$

4. Suppose that θ and γ are positive integers, where $\theta < \gamma$.
If $\frac{\theta+\gamma}{2} : \sqrt{\theta\gamma} = 13:12$, determine the least value of γ.

設 θ 及 γ 為正整數，當中 $\theta < \gamma$。若 $\frac{\theta+\gamma}{2} : \sqrt{\theta\gamma} = 13:12$，求 γ 的最小值。

$\gamma =$

Hong Kong Mathematics Olympiad (2017/18)

Final Event 4 (Group)

FOR OFFICIAL USE

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			+ Bonus score				Time	Min. / Sec.
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除非特別聲明，答案須用數字表達，並化至最簡。

1. Let $X=\sqrt{2018-\sqrt{A}}$ be a positive integer. Determine the largest value of A.
設 $X=\sqrt{2018-\sqrt{A}}$ 是正整數。求 A 的最大值。

$A =$

2. Determine the value of B, the product of all the real roots of $(12x-1)(6x-1)(4x-1)(3x-1)=5$.
求方程 $(12x-1)(6x-1)(4x-1)(3x-1)=5$ 的所有實根之乘積 B 的值。

$B =$

3. Determine the value of
$$C=\cos\frac{\pi}{15}\times\cos\frac{2\pi}{15}\times\cos\frac{3\pi}{15}\times\cos\frac{4\pi}{15}\times\cos\frac{5\pi}{15}\times\cos\frac{6\pi}{15}\times\cos\frac{7\pi}{15}.$$
求 $C=\cos\frac{\pi}{15}\times\cos\frac{2\pi}{15}\times\cos\frac{3\pi}{15}\times\cos\frac{4\pi}{15}\times\cos\frac{5\pi}{15}\times\cos\frac{6\pi}{15}\times\cos\frac{7\pi}{15}$ 的值。

$C =$

4. Let r, s and t be positive real numbers with $r^2+s^2+t^2=rs+st+rt$. If $r=1$, determine the value of $D=s+t$.
設 r, s and t 是正實數，且 $r^2+s^2+t^2=rs+st+rt$。若 $r=1$，$D=s+t$ 的值。

$D =$